

Concluding remarks

A progression of great thinkers has moved the human race towards the adoption, first of an economical and efficient number system containing zero and based on place value, and then of a universal algebra, APL, which operates on arrays or multiple quantities, and is totally devoid of words.

There have also been those who resisted the inevitable progress, who found it difficult to adopt new and improved tools for thought. In our own time we hear appeals to revert from this high intellectual level and use English words, and to submit to the tyranny of scalars, as if Sylvester's eloquence a century ago had fallen on deaf ears.

Unlike its predecessors, APL is an executable notation. APL represents, in a phrase used by Babbage, the "triumph of symbols over words." As so many of our distinguished predecessors predicted, it makes reasoning easier. APL is the result of brilliant insight, careful thought, and hard work through at least 5000 years. Iverson is the latest in a succession that includes Peano, Sylvester, Cayley, De Morgan, Boole, Newton, Leibniz, Napier, Stevinus, Fibonacci, Diophantus, and the unknown Egyptian whose work was copied by Ahmes the scribe.

In 1866 Sylvester proclaimed that: "To attain clearness of conception, the first condition is 'language,' the second 'language,' the third 'language'—Protean speech—the child and parent of thought."¹²⁴

In reflecting on the significance of APL I have adopted a historical approach. Having done so I find that Sylvester had something to say on that subject also. The occasion was his Presidential Address to the British Association¹²⁵ in 1869 when he said: "the relation of master and pupil is acknowledged as a spiritual and lifelong tie, connecting successive generations of great thinkers with each other in an unbroken chain."

We think in a different way because of APL.

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