where new arbitrary signs are invented to denote operators. . . . The more complicated the enquiries on which we enter, and the more numerous the quantities which it becomes necessary to represent symbolically, the more essentially necessary it will be found to assist the memory by contriving such signs as may immediately recall the thing which they are intended to represent."[119]

Sylvester, in 1877, said "It is the constant aim of the mathematician to reduce all his expressions to their lowest terms, to retrench every superfluous word and phrase, and to condense the Maximum of meaning into the Minimum of language."[120]

Whitehead, in 1911, claimed that "By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and in effect increases the mental power of the race. . . . By the aid of symbolism we can make transitions in reasoning almost mechanically by the eye, which would otherwise call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copy-books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them."[121]

Bertrand Russell said: "The great master of the art of formal reasoning, among men of our own day, is an Italian, Professor Peano, of the University of Turin. He has reduced the greater part of mathematics (and he or his followers will, in time, have reduced the whole), to strict symbolic form, in which there are no words at all."

In the first paragraph of his book in 1959, Russell wrote: "There is one major division in my philosophical work: in the years 1899–1900 I adopted the philosophy of logical atomism and the technique of Peano in mathematical logic. This was so great a revolution as to make my previous work, except such as purely mathematical, irrelevant to everything I did later. The change in these years was a revolution; subsequent changes have been of the nature of an evolution."[122]

And finally, Giuseppe Peano himself, in his paper on "The Importance of Symbols in Mathematics" in 1915 wrote: "The oldest symbols, which are also the most used today, are the digits used in arithmetic, which we learned about 1200 from the Arabs, and they from the Indians, who were using them about the year 400. The first advantage that one sees in the digits is their brevity. . . . Further reflection reveals that these symbols are not just shorthand, i.e., abbreviations of ordinary language, but constitute a new class of ideas. . . . The use of digits not only makes our expressions shorter, but makes arithmetical calculation essentially easier, and hence makes certain tasks possible, and certain results obtainable, which could not otherwise be the case in practice. For example, direct measure assigned to the number Pi, the ratio of the circumference of a circle its diameter, the value 3. . . .

"Archimedes, about 200 B.C., by inscribing and circumscribing polygons about a circle, or rather by calculating a sequence of square roots, using Greek digits, found Pi to within 1/500. The substitution of Indian digits for the Greek allowed Aryabhata, about the year 500, to extend the calculation to 4 decimal places, and allowed the European mathematicians of 1600 to carry the calculation out to 15 and then 32 places, still following Archimedes' model. Further progress, i.e., the calculation of 100 digits in 1700, and the modern calculation of 700, was due to the introduction of series.

"The same thing may be said for the symbols of algebra. . . . Algebraic equations are much shorter than their expression in ordinary language, are simpler, and clearer, and may be used in calculations. This is because algebraic symbols represent ideas and not words. . . . Algebraic symbols are much less numerous than the words they allow us to represent.

"The evolution of algebraic symbolism went like this: first, ordinary language; then, in Euclid, a technical language in which a one-to-one correspondence between ideas and words was established; and then the abbreviation of the words of the technical language, beginning about 1500 and done in various ways by different people, until finally one system of notation, that used by Newton, prevailed over the others.

"The use of algebraic symbols permits schoolchildren easily to solve problems which previously only great minds like Euclid and Diophantus could solve. . . . The symbols of logic too are not abbreviations of words, but represent ideas, and their principal utility is that they make reasoning easier. All those who use logical symbolism attest to this."[123]