Figure 11 Gibbs's example of transformation

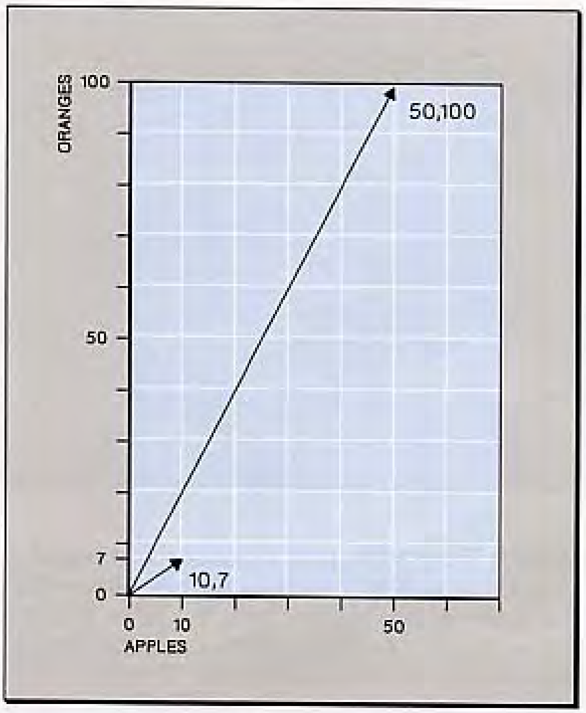

Figure 11 illustrates the problem Gibbs posed and makes the answer obvious. Although Gibbs did not turn to Hamilton, Sylvester, or Cayley for the solution, I betray their influence in Figure 12, where I separate the versor (as a rotation matrix) and the tensor (a scalar). The example can be worked as follows:

The transformation matrix (with tensor and versor composed):

```
      X← 50 100 ⌹ 10 ¯7,[¯0.5] 7 10
      N← (1 ¯1 × X),[¯0.5] ⌽X
      N+.× 10 7
 50 100
```

Isolate the tensor and determine the angle of rotation in degrees:

```
      ⎕← Y← (+/ X*2)*0.5
9.16

      (180÷○1)× ¯2 ¯1 ○ X÷Y
28.44 28.44
```

Confirm by composing the tensor and versor, where RFD is Radians From Degrees:

```
      ∇Z←RFD X
[1]    Z←○X÷180
[2]   ∇

      ∇Z←F X
[1]    Z←2 2⍴1 ¯1 1 1×2 1 1 2○RFD X
[2]   ∇

       (9.16 × F 28.44) +.× 10 7
50 100
```

The wondrous tale of multiple quantity

This example, simple though it is, throws light upon the nature of the "new world of thought" to which Sylvester gave the name of "Universal Algebra or the Algebra of multiple quantity" in 1884.

James Joseph Sylvester was born in 1814. In 1837 he completed his studies at Cambridge and published the first of his 342 papers. It was on crystallography. His next two papers were on the motion of fluids and rigid bodies—all topics of importance to my own subject of geology—and all amenable to matrix algebra. Some additional history can be found in Reference 102.

Sylvester,[103–106] the self-styled mathematical Adam, gave "more names (passed into general circulation) to the creatures of mathematical reason than all the other mathematicians of the age combined" (1888). In 1850, the year he was called to the bar, he introduced the term *matrix* for "a rectangular array of terms, out of which different systems of determinants may be engendered as from the womb of a common parent."[107,108] Sylvester introduced[109] the Greek letter *lambda* (λ) for the latent roots of a characteristic equation (his terms) in 1852—three-quarters of a century before the term *eigenvalue* was invented; and in 1853 he introduced the *inverse matrix*.[110]

In 1884, at the age of 70, he published his Lectures on the Principles of Universal Algebra, the "apotheosis of algebraical quantity," in the *American Journal of Mathematics*, which he himself founded and edited. His title reminds us that Newton used the term *universal arithmetic* for what we call *algebra*. Emphasizing the importance of matrices as multiple quantity, he speaks of a second birth of algebra, its *avatar* in a new and glorified form.[111] In the words of this enthusiast, who lived a century before APL was implemented: "A matrix of quadrate