lished on the rotation of a solid body. These are all topics that led naturally to the algebra of matrices. A matrix can, as we know, be looked upon as an array of multidimensional vectors, and so it is interesting that in 1843, the year Hamilton discovered quaternions, Cayley published on "the Geometry of (n) dimensions." Work on matrices was almost bound to follow.

Cayley was much influenced by Hamilton and visited Hamilton in Dublin. Cayley wrote his first paper on quaternions in 1845 at the age of 24, and considered the quaternion theory to be "a generalization of the analysis which occurs in ordinary Algebra." Later the same year he wrote on "The octuple system of imaginaries," showing that consistent arithmetics exist for couples, quadruples (but not triplets), and eight-fold hypercomplex numbers. Two years later he demonstrated that "in the octuple system of imaginary quantities neither the commutative nor the distributive law holds."

In 1848 Cayley showed that the combined effect of two rotations could be represented as the product of two quaternions, and shortly afterwards Sylvester (in the year he introduced the term *matrix*) pointed out that any number of rotations can be represented by a single rotation about one axis. As we would now say: each rotation can be represented by a matrix, and the product of these matrices is a matrix completely describing the combined rotation, whose axis is an eigenvector of this matrix, and the angle of rotation can be found from the corresponding eigenvalue. By 1855 Cayley used matrix product (calling it the *composition* of matrices), and in his memoir of 1858 he wrote: "It will be seen that matrices comport themselves as single quantities; they may be added, multiplied, or compounded together, etc.: the law of the addition of matrices is precisely similar to that for the addition of ordinary algebraical quantities; as regards their multiplication (or composition), there is the peculiarity that matrices are not in general convertible; it is nevertheless possible to form the powers (positive or negative, integral or fractional) of a matrix . . . "[17] In this memoir he uses Sylvester's latent roots (eigenvalues), but without naming them.

Sylvester's paper, written in 1882, begins thus: "Professor Sylvester referred to the general question of representing the product of sums of two, four, or eight squares under the form of a like sum, and mentioned that Professor Cayley had been the first

Figure 8 Sylvester's locative symbols

$$\theta + 8h + 8t + 2u$$

is 1882

$$\rho = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \lambda & \mu \\ \nu & \pi \end{pmatrix}$$

$$\rho = a\lambda + b\mu + c\nu + d\pi$$

to demonstrate, by an exhaustive investigation, the impossibility of extending the law applicable to 2, 4, and 8 to the case of 16 squares. The new kind of so-called imaginaries referred to by Professor Cayley are, as far as Mr. Sylvester is aware, the first example of the introduction into Analysis of *locative symbols* not subject to the strict law of association, and he considers the law regulating the connexion of the two products represented by a succession of three such symbols, most interesting, inasmuch as such products are either identical, or if not identical, of the same absolute value, but with contrary signs: most persons, before this example had been brought forward, would have felt inclined to doubt the possibility of locative symbols ('vulgo' imaginary quantities) whose multiplication table should give results inconsistent with the common associative law, being capable of forming the groundwork of any real accession to algebraical science . . . . "[91]

His footnote is illuminating (compare also Reference 92): "Using θ, h, t, u to denote thousands, hundreds, tens, units, the year of grace in which we live may be represented by $\theta + 8h + 8t + 2u$ [—] θ, h, t, u, being locative symbols which it would be absurd to style 'imaginary quantities'; but they are as much entitled to that name as the i, j, k, or any like set of symbols—the only essential difference being that one set of symbols is limited, the other unlimited in number—and accordingly the law of combination of the one set is given by a finite and of the other an infinite 'multiplication table' . . . The 'locatives' indicate out of what 'basket,' so to say, the 'quantities' appearing in an analytical expres-