forces.[79–81] The discovery is so important that Newton[82] stated it as Corollary I immediately after his Laws of Motion. Authors of modern textbooks often suggest that the rule for vector addition is quite arbitrary by saying that the sum of two vectors is defined to be a third vector whose components are given by the sum of the corresponding components of the given vectors. Such a statement disguises the fact that in the real world we observe that forces combine in this manner.

Many first encounter the word *vector* in Kepler's so-called Second Law of Planetary Motion: the radius vector sweeps out equal areas in equal times. Kepler's prodigious calculations are even more remarkable when we remember how few mathematical symbols were available—logarithms, and even the decimal point had not yet been invented.

Once Kepler had found a mathematical relationship that held throughout space, he looked for a deeper reason. Introducing the *Newtonian* concept of force into science, he claimed that a magnetic force (*anima motrix*) emanated from the sun and carried the planets in their orbits.[83]

Vector is the Latin word for a carrier, and it is used in medicine today in this sense. *Vector meus* is "my horse," and *vehicle*, *wagon*, *way*, and *convection* are from the same root. It was therefore an appropriate word for whatever it is that carries the planets in their orbits round the sun. I looked in vain for it in Kepler,[84] but Small[85] gives *radii vectores*. Harris, in 1704, defines *vector* to be "A line supposed to be drawn from any Planet moving round a Centre, or the Focus of an Ellipse, to that Centre or Focus, is by some writers of the New Astronomy, called the Vector; because 'tis that line by which the Planet seems to be carried round its Centre."[86]

A vector in two dimensions can be represented by a complex number (and vice versa). Wessel, a Norwegian surveyor, was the first to realize this, but his work, though published in 1799, was unrecognized until 1897. A modern geometric treatment of the addition and multiplication of complex numbers was given by Argand in 1806, but these ideas received little attention until Gauss took up the topic in 1831.

If complex numbers can represent points in a plane, it is natural to try to create hypercomplex numbers to represent points in three-dimensional space. Sir William Rowan Hamilton finally succeeded in doing this in 1843.[87]

In a long paper on "algebraic couples" written in 1837 Hamilton said: "In the THEORY OF SINGLE NUMBERS, the symbol $\sqrt{-1}$ is 'absurd' [it is an impossible root, or an imaginary number]; but in the THEORY OF COUPLES, the same symbol $\sqrt{-1}$ is 'significant' [i.e., it denotes a possible root, or a real couple]." What did he mean? I found the answer more clearly in Hamilton's own words than in modern textbooks.

Knowing that if you double a force you double the vector that represents it, Hamilton looked on *2 times* as the operator that doubles; it is a special case of what he called a *tensor*, an operator that stretches (not to be confused with the modern use of the word). In the same way *−1 times* is a reversor. Moreover if *$\sqrt{2}$ times* is applied twice it doubles; and if *$\sqrt{-1}$ times* is applied twice it reverses. Consequently *i times* (where i is $\sqrt{-1}$) is a versor, or operator that rotates a vector without changing its length; it is taken as producing a counter-clockwise rotation of 90 degrees. Application of *−2i times* would then be the composition of a rotation, a stretch, and a reversal. It is to Hamilton that we owe our terms *scalar* and *vector* (1846).

It seemed plausible that if couplets represent vectors in two dimensions, triplets would represent vectors in three dimensions, but after years of unsuccessful attempts, Hamilton realized, in a flash of genius, that a consistent algebra of triplets is impossible. Four terms (quaternions) are needed, shown in the example below:

complex: $a + bi$
$i^2 = {}^-1$

quaternion: $a + bi + cj + dk$
$i^2 = j^2 = k^2 = ijk = {}^-1$
$ij = -ji$

Quaternions are of interest to the pure mathematician because they do not obey the laws of ordinary arithmetic: multiplication of quaternions is associative but not commutative.

Hermann Grassmann (a German schoolmaster) worked on vector systems at about the same time as Hamilton, and it was Grassmann who, in 1862, gave us *inner* and *outer products*, analogous to the scalar and vector parts of Hamilton's multiplication of quaternions.[88–90]

All of Arthur Cayley's early papers were on, or used, determinants, and both he and Sylvester pub-