

Unlike the play on words in Lewis Carroll's *Through the Looking Glass*, the distinctions between *zero*, *emptiness*, and *nothing* are not only useful but essential. The recursive APL functions already given include in a single line, *zero*, an *empty vector*, and (when the end condition obtains) *nothing*.

Logic

Because logic deals with two states, true and false, the mathematics of 0 and 1 is said to be logical. Propositions, or statements that may be judged true or false, are logical statements, and computers are logical machines because they manipulate binary digits (bits). The mathematics of logic began with Boole,⁵⁸⁻⁶¹ just at the time Sylvester introduced the term *matrix*. Jevons considered Boole's work to be, perhaps, "one of the most marvellous and admirable pieces of reasoning ever put together."⁶² Bertrand Russell thought highly of Boole's work, going so far as to claim that "Pure mathematics was discovered by Boole in a work which he called 'The Land of Thought.'⁶³

"Let us conceive, then," wrote Boole, "of an Algebra in which the symbols x , y , z , etc. admit indifferently of the values of 0 and 1, and of these values alone."⁶⁴ Today we call a vector consisting of 1s and 0s a logical or Boolean vector, and Iverson notation, from its outset, used Boolean vectors to select from arrays, whether or not they were logical.¹ Where Boole used $x(s)$ to stand for the selection of all the x s from subset s , Iverson used u/s in APL (or $u\#s$ in J), which is compression if u is Boolean and replication if it is not.

Because a computer's memory and registers can be described as arrays of 1s and 0s, we now recognize that Boole laid the foundation for the design and description of modern computers—which are logical machines. But to most of his contemporaries his work seemed of little significance. The obituary notice in *The Athenaeum* (December 17, 1864) dryly reported that "The Professor's principal works were 'An Investigation into the Laws of Thought,' and 'Differential Equations,' books which sought a very limited audience, and we believe found it."

The Oxford English Dictionary cites the use of *Boolian algebra* [sic] in 1895 and 1902, but however we spell it, the usage is questionable. As Sylvester emphasized, there is only one universal algebra, which must, of course, include logic: "I have also a great

repugnance to being made to speak of Algebras in the plural; I would as lief acknowledge a plurality of Gods as of Algebras."⁶⁵ I am sure he would have approved of APL, which incorporates logical functions so that they can be used together with arithmetic functions in a single expression. For example, from Iverson:⁶⁶

"A theorem is a proposition which is claimed to be universally true, i.e., to have the value 1 when applied to any element in the universe of discourse. For example, the proposition

$$((0=2|X)\wedge(0=3|X))\leq 0=6|X$$

is a theorem which may be verbalized in a variety of ways:

" X is divisible by 2 and X is divisible by 3 implies that X is divisible by 6.

"Any number divisible by both 2 and 3 is also divisible by 6.

"If X is divisible by both 2 and 3 then X is divisible by 6.

"Divisibility by 2 and 3 implies divisibility by 6."

According to John Venn (whose name is well known in connection with the diagrams that so effectively illustrate the meanings of *and*, *or*, and *not*), Jevons "was certainly the first to popularize the new conceptions of symbolic logic." The boldness, originality, and beauty of Boole's system fascinated him, and Jevons's book⁶⁷ was largely founded on Boole. Jevons, unlike Boole, emphasized the importance of the *inclusive or* and his symbol $(\cdot|)$ survives (though without the dots) in PL/I and in countless IBM technical manuals.

In 1865, Jevons completed construction of his reasoning machine, or logical abacus, adapted to show the workings of Boole's logic in a half mechanical manner, a full account of which was published by the Royal Society in 1870.⁶⁸ Mechanical devices had been designed by Napier, Pascal, Thomas of Colmar, and in Jevons's own time by Babbage, Stanhope,⁶⁹ and Smee,^{70,71} but Jevons claimed that until the work of Boole, logic had remained substantially as molded by Aristotle 2200 years ago. De Morgan, whose *Formal Logic*⁷² was published, by coincidence, on the same day as Boole's book,⁵⁸ pointed to the connection between two revealing facts: "logic