Though the ancient Egyptians used *heap* as a general term for an unknown quantity,[46,47] Diophantus, a Greek mathematician in Alexandria about 300 AD, was probably the original inventor of an algebra using letters for unknown quantities.[48] Diophantus used the Greek capital letter *delta* (not for his own name!) for the word *power* ("*dynamis*"; compare "dynamo," "dynamic," and "dynamite"), which is therefore one of the oldest terms in mathematics.[14] Today we use a conjunction to raise a function to a power. The syntax brings out the parallelism between raising a number to a power and applying a function an equal number of times. The algorithm fails when the number of doublings is further increased.[49]

Hindu-Arabic numerals and zero

Hindu-Arabic numerals were introduced to the western world by Leonardo of Pisa (Fibonacci) in 1202 with these words: "*Novem figure Indorum he sunt 9 8 7 6 5 4 3 2 1. Cum his itaque nouem figuris, et cum hoc signo 0, quod arabic cephirum appellatur, scribitur quilibet numeros.*" [The nine numerals of the Indians are these: 9 8 7 6 5 4 3 2 1. With them and with this sign 0, which in Arabic is called cipher, any desired number can be written.[50]] (Slightly different in Reference 51.)

It was, however, far easier for most people to add and subtract with Roman numerals (or with Egyptian hieroglyphics for that matter), and this was sufficient for their needs. They also believed that, with the new system, accounts could be more easily falsified—for instance by changing zero into 6 or 9. Adoption of the new symbols was therefore very slow. The oldest known Hindu-Arabic numerals on a gravestone are dated 1371, and their earliest use on coins outside Italy was in 1424. They were not used on an English coin until 1551.[52] Even today Roman numerals are used for royalty. Clocks not powered by digital technology still commonly display old-style symbols on their dials.

As long as calculations were performed on *counters* or *boards* (see the etymology of *bank* and *bankrupt*) there was no need for a symbol to show an *empty* column. Menninger has some excellent sentences on the subject: "Zero is something that must be there to show that nothing is there, [for] only the abstract place-notation needs zero. Zero first liberated the digits from the counting board."[53]

Surely one of the most remarkable inscriptions in Europe[54] is: I · V^c · V . It records the date 1505 in symbols which, though Roman, are used with a positional significance unknown in Rome. The scribe "had heard about the new place-value system and now tried to find it in the Roman numerals. Since the meaning of the zero was still not clear to him, I V 0 V = 1505; at the critical point he yielded and retreated into the 'named' place-value notation."[55] He solved his problem by inserting a superscript letter *c* to identify the hundreds column (compare Sylvester's *locative symbols*). It is exciting to catch the conversion from the old way to the new as it was happening!

If it took so long for Hindu-Arabic numerals to make their way in the western world, we can hardly expect APL to be universally adopted in 25 years. But we can find encouragement in Menninger's words: "These ten symbols which today all peoples use to record numbers, symbolize the world-wide victory of an idea. There are few things on earth that are universal, and the universal customs which man has successfully established are fewer still. But this one boast he can make: the new Indian numerals are universal."[56]

One of the satisfactions in working with APL comes from its consistency and completeness, exemplified by its recognition of *identity elements*, i.e., arguments that, used with a dyadic function, give a result identical to the other argument. If at each iteration in a FORTRAN loop, we accumulate by adding to a variable named SUM, why must we set SUM to zero before entering the loop? The reason is that zero is the identity element for addition, as 1 is of multiplication. APL, being rich in scalar dyadic functions, needs more kinds of identity elements than other languages do.

Although the computation of pi by inscribed polygons is recursive, we did not accumulate intermediate results, but proceeded at once to the next approximation. On the other hand, Sylvester's algorithm for Egyptian unit-fractions constructs a vector, and the starting point must therefore be an empty vector.

We can calculate interest payments on a declining balance by following the same recursive paradigm.

Ordinary APL:

```
     ∇Z← A where W
[1]    Z←A
[2]  ∇
```