Figure 6 Rhind Papyrus problem 32; multiplying 12 by 12

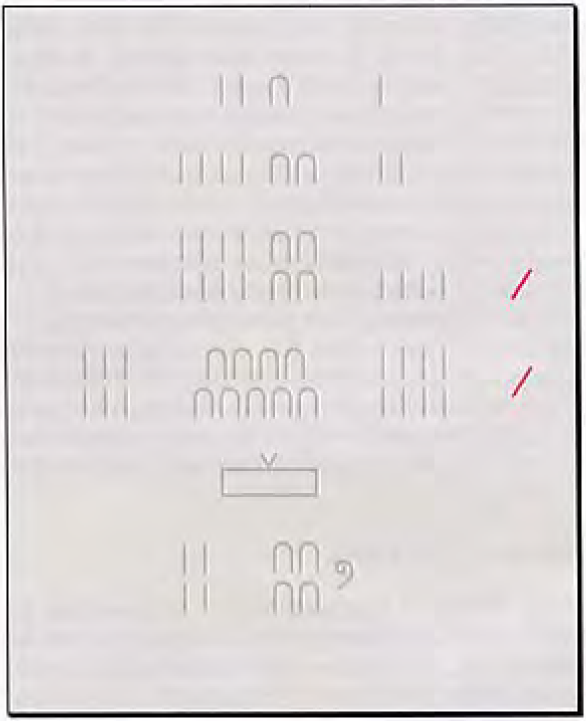
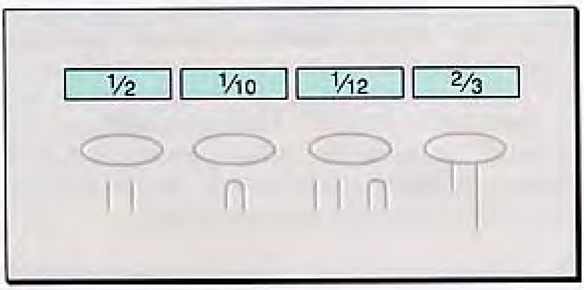


Figure 7 Hieroglyphic method of showing fractions



IBM's System/360* and its descendants use this ancient method to multiply integers. Microcode for fixed-point multiplication builds the 1×, 2×, 3×, and 6× products of the multiplicand in local storage. Then, just as the scribes did nearly 4000 years ago, it combines the products corresponding to the multiplier. If the multiplier is 8 or more, a shift of 4 is first made (corresponding to multiplication by 16), and then products are subtracted rather than added; e.g., to multiply by 11, first shift to multiply by 16, then subtract 6× and add 1×. One may ask why the products used by System/360 are 1×, 2×, 3×, and 6× instead of the 1×, 2×, 4×, 8× used by the Egyptians. When I raised this question in a lecture in New York in 1982, John Macpherson (who was the first to implement binary coded decimal on an IBM computer) gave me the explanation in engineering terms.

However unfamiliar its symbols may be to us, the hieroglyphic message is inherently simple. So it is with the symbols of APL, all of which stand for well-known or easily understood operations. Many today, as Oughtred found 350 years ago, are "scared by the newness of the delivery; and not by any difficulty in the thing itself"!

The ancient Egyptians used mathematics for practical purposes, such as paying wages and collecting taxes. Consider the instructive example of salary distribution at the Temple of Illahun—not paid in salt (as the word "salary" implies) but in jugs of beer and loaves of bread. Division, of course, often produces fractions, and the hieroglyphic way to represent fractions can be seen in Figure 7.

All fractions were represented as unit fractions, i.e., with a numerator of 1. Even 2/3, which seems like an exception, was represented as the unit fraction 1/1.5. The eye-like symbol is perhaps the earliest of all APL function symbols. It is the reciprocal, or *monadic divide*, which in APL has become an eye closed into a slit, with dots above and below (÷).

If a loaf of bread is divided into 10 parts, and you are to get 1 share, your portion is 1/10; if you are to get 2 shares your portion is 1/5; and if you are to get 5 shares your portion is 1/2. From these simple fractions, other shares can be computed by combination.[42] For example, 3 shares are the same as 1 + 2 shares, i.e., 1/5 + 1/10; 4 shares are the same as 2 + 2 shares, i.e., 1/5 + 1/5, which, by consulting a table of values of 2/n, is set down as 1/3 + 1/15.

Sylvester became interested in the unit fractions of the Egyptians when reading "the chapter in Cantor's *Geschichte der Mathematik* which gives an account of the singular method in use among the ancient Egyptians for working with fractions. It was their curious custom to resolve every fraction into a sum of simple fractions according to a certain traditional method, not leading, I need hardly say, except in a few of the simplest cases, to the expansion under the special form to which I have the name of a fractional sorites."[43]