Figure 5 Exponents enclosed in circles (Stevinus, 1585)

$3x^2 - 5x + 1$	3② − 5① + 1⓪
34.735	34⓪ 7① 3② 5③

resenting related monadic and dyadic functions. Iverson simplified syntax by abandoning function hierarchy (originally imposed for writing polynomials) and making each function take everything to its right as its right argument.

Acceptance of good symbols has, however, never been easy. After introducing the *times* symbol (Saint Andrew's cross) in 1631, Oughtred wrote: "This manner of setting downe Theoremes, whether they be Proportions, or Equations, by Symboles or notes of words, is most excellent, artificiall, and doctrinall [i.e., serving to teach]. Wherefore I earnestly exhort every one, that desireth though but to looke into these noble Sciences Mathematicall, to accustome themselves unto it: and indeede it is easie, being most agreeable to reason, yea even to sence. And out of this working may many singular consectaries [i.e., conclusions] be drawne: which without this would, it may be, for ever lye hid."[11]

But 15 years later, still more encouragement was needed: "[My] Treatise being not written in the usuall synthetical manner, nor with verbous expressions, but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newness of the delivery; and not any difficulty in the thing it selfe. For this specious [i.e., pleasing to the eye] and symbolicall manner, neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation."[11]

It seems that not much has changed, judging from the experience of Giuseppe Peano (who provided two of APL's symbols). We are told that he "used a great deal of symbolism because he wished to sharpen the reasoning. . . . Peano used this symbolism in his presentation of all of mathematics, notably in his Formulario mathematico (5 vols., 1895–1908). He used it also in his lectures, and his students rebelled. He tried to satisfy them by passing all of them, but that did not work, and he was obliged to resign his professorship at the University of Turin."[12]

Smith, quoting Nesselmann's *Algebra of the Greeks* (1842), says that mathematics evolves through three stages: *rhetorical*, with words and sentences in full; *syncopated*, in which words are condensed by abbreviation; and *symbolic*, in which there are no words at all.[13,14] Consider the way we write equations. Comparison of 20 examples from 1463 to 1693[15] shows how long it took to pass from words to our present symbolic system. Simon Stevin (Stevinus, 1548–1620), for instance, made great progress by identifying exponents, writing them enclosed in circles (See Figure 5). His books (1585, 1586) were influential in promoting the use of the new methods. (See Reference 16.)

The superscript method of denoting a to the power b (that is, a^b) was used by Hume in 1636, though his use of Roman numerals for the exponent shows he thought only of integer powers. The form we use now was first used by Descartes in 1637. John Wallis, a distinguished predecessor of Sylvester's as Savilian Professor of Geometry in Oxford, was one of the first to write equations in the form we use today, though even he often wrote *aaaa* for a^4. Until the end of the eighteenth century it was, indeed, common practice to write *aa* for a^2. Wallis, who gave us our symbols for *greater-than-or-equal-to* (≥) and *less-than-or-equal-to* (≤) and our symbol for infinity (∞), found a meaning for negative exponents (1655, 1657), but Newton was the first to permit the exponent to be positive, negative, integer, or fractional (1676).

Euler, in 1777, introduced the symbol i (impossible or imaginary) for √¯1, and by 1837 Sir William Rowan Hamilton had so adopted the geometrical interpretation of complex numbers (Wessel, Gauss, Argand) that it could be said that exponentiation had been extended to the case of a negative number with a fractional exponent. Cayley further extended the scope of exponentiation by raising matrices to positive integer powers and to the power ¯1, which he called the "inverse or reciprocal" matrix.[17,18] Today's APL handles all these cases directly.

To indicate that a word was abbreviated, the practice used to be to put a stroke (solidus) through the last letter. This accounts for the lines still seen in