

Figure 4 First appearance of symbols in print

Plus, Minus	+ -	Johann Widman	1489
Equals	=	Robert Recorde	1557
Times	×	William Oughtred	1631
Greater than, Less than	> <	Thomas Harriot	1631
Exponentiation	$A^{II}$	James Hume	1636
Greater than or equal to, Less than or equal to	$\geq \leq$	John Wallis	1655
Divide	÷	Johann H. Rahn	1659
Summation	$\Sigma$	Leonard Euler	1755
Factorial	!	Christian Kramp	1808
Absolute Value	A	K. Weierstrass	1841
Membership	$\epsilon$	Giuseppe Peano	1889
Logical Not	$\sim$	Giuseppe Peano	1893
Inclusive Or	$\vee$	Whitehead and Russell	1909
And	$\wedge$	Alfred Tarski	1933

to become a Fellow of the Royal Society (as president of the society, Pepys gave his imprimatur to Newton's "Principia"). As Secretary of the Navy he became one of the nation's leading financiers.

How seldom do we look back in maturity at what we learned by rote as children, and that is why I like the title (as well as the content) of Klein's *Elementary Mathematics from an Advanced Standpoint*.<sup>6</sup> We are taught as if the common mathematical symbols came to humankind in antiquity engraved on stone; as if they had no history. The dates when some of these symbols first appeared in print show that our notation evolved over centuries<sup>7,8</sup> (see Figure 4). The imprints on our bank checks show that in our own time technology has changed some of our familiar symbols.

### The acceptance of symbols

The symbol for *plus* is probably an abbreviation for the Latin *et*, and that for *minus* may be "a simple bar used by merchants to separate the indication of the tare, for a long time called 'minus,' from that of the total weight of the merchandise."<sup>9</sup> De Morgan thought the symbols might be marks on sacks or barrels showing whether they were over or under weight. Recorde, in 1557, first used these signs in an English book, the same one in which he gave us the

*equals* symbol, which he chose "because noe 2 thynges can be moare equalle." Euler's  $\Sigma$  (sigma) suggests *summation*; epsilon is the first letter of the Greek *esti* (is a), which suggests *membership*; and the symbol for *or* is the first letter of the Latin *vel*.

In his survey of the development of mathematics, Kline pointed out that Leibniz "certainly appreciated the great saving of thought that good symbols make possible. Thus by the end of the seventeenth century, the deliberate use of symbolism—as opposed to incidental or accidental use—and the awareness of the power and generality it confers entered mathematics."<sup>10</sup>

Our notation having been at least 500 years in the making, it is no surprise that the story is not yet at an end. What is remarkable is that Iverson is apparently the first to look at the consistency and completeness of the notation as a whole. Function syntax is inconsistent; e.g., *summation* has its argument to the right, *factorial* to the left, and *absolute value* is written on both sides of its argument. *Exponentiation* has no symbol at all; its second argument is merely written as a superscript. Iverson also considered which other functions have sufficient utility to warrant separate graphic symbols. He showed that function names should not be elided, and pointed out the advantage of each symbol rep-