

Language as an intellectual tool: From hieroglyphics to APL

by D. B. McIntyre

We learn elementary mathematics before understanding the source of its symbols and procedures, which therefore appear, incorrectly, to have been decreed ready-made. Language and reason are intimately related, and the embodiment of an idea in a symbol may be essential to its comprehension. APL unifies algebra into a single consistent notation; it allows us to exploit the powerful concepts of functions and operators; and it helps us to escape from the tyranny of scalars by giving us the tools to think in terms of arrays, or multiple quantity, as J. J. Sylvester so eloquently urged us to do a century ago. APL has an intellectual consistency that is a source of satisfaction and pleasure. This paper traces the history of symbols from hieroglyphics to APL.

The APL language, a language with symbols and not words, is one of the intellectual triumphs of our time. Its modern incarnation began with Iverson notation,^{1,2} but its roots go far back into the past.

In the beginning

Perhaps the earliest record of what came to be APL was carved on a sculptured mace of granite about 3100 BC, before the invention of papyrus. Of course you cannot read it, unless as is the case with contemporary APL, you know the meaning of the symbols.

We shroud in mystery whatever we do not understand. In crystal optics we speak of "extra-ordinary" rays, though there is, of course, nothing extra-ordinary about them. Negative numbers were called absurd or fictitious. Even after Leonardo of Pisa (known as Fibonacci), in the year 1202, had taught us to recognize debt as a negative asset, it took another 400 years before the number scale was represented geometrically. Intellectual progress is slow, and an additional 250 years passed before Sylvester showed how absurd it was to style as imaginary the quantities represented by the symbols i , j , k of "complex" numbers and quaternions.

I remind you of the words of Whitehead: "Mathematics is often considered to be a difficult and mysterious science, because of the numerous symbols which it employs. Of course, nothing is more incomprehensible than symbolism we do not understand."³

The inscription illustrated in Figure 1 is a record of the triumph of Menes, founder of the first dynasty

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